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MATHEMATICS
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PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Monday 15 November 2010 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

3 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 7]*

Find $\lim_{x \to 0}\left(\frac{1-\cos x^6}{x^{12}}\right)$.

2. *[Maximum mark: 16]*

Determine whether or not the following series converge.

(a) $\sum_{n=0}^{\infty}\left(\sin\frac{n\pi}{2}-\sin\frac{(n+1)\pi}{2}\right)$ *[3 marks]*

(b) $\sum_{n=1}^{\infty}\frac{e^n-1}{\pi^n}$ *[7 marks]*

(c) $\sum_{n=2}^{\infty}\frac{\sqrt{n+1}}{n(n-1)}$ *[6 marks]*

3. *[Maximum mark: 9]*

(a) Using the Maclaurin series for the function e^x, write down the first four terms of the Maclaurin series for $e^{-\frac{x^2}{2}}$. *[3 marks]*

(b) Hence find the first four terms of the series for $\int_0^x e^{-\frac{u^2}{2}}\,du$. *[3 marks]*

(c) Use the result from part (b) to find an approximate value for $\frac{1}{\sqrt{2\pi}}\int_0^1 e^{-\frac{x^2}{2}}\,dx$. *[3 marks]*

4. *[Maximum mark: 13]*

Solve the differential equation

$$(x-1)\frac{\mathrm{d}y}{\mathrm{d}x}+xy=(x-1)\,\mathrm{e}^{-x}$$

given that $y=1$ when $x=0$. Give your answer in the form $y=f(x)$.

5. *[Maximum mark: 15]*

Consider the infinite series

$$\frac{1}{2\ln 2}-\frac{1}{3\ln 3}+\frac{1}{4\ln 4}-\frac{1}{5\ln 5}+\ldots\,.$$

(a) Show that the series converges. *[4 marks]*

(b) Determine if the series converges absolutely or conditionally. *[11 marks]*